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Investigation of influence of Kerr and detuning parameters over instability gain in the three wave resonant interaction of stimulated Raman backscattering

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Abstract
In this article, we study the modulation instability that rules over the evolution of three wave resonant interaction of backward stimulated Raman process. We intend to investigate the influence of Kerr and detuning parameters over the conventional instability gain under both the anomalous and normal dispersion regimes. We also tend to reveal the inertial instability gain arising in the absence of dispersion and explore its dependence over the system parameters.

Keywords Stimulated Raman backscattering · Modulation instability · Anomalous and normal dispersion regimes

1 Introduction

In today's information age, the advent of the mighty fiber optics technology has quenched the thirst for fast, accurate and high capacity communication. The nonlinear interactions of intense light beams with the molecules of the fiber optic transmission medium lead to various fascinating physical effects which have opened up a large field of potential applications in optical communication systems. One such effect is the stimulated Raman scattering, the inelastic process, which involves the interaction of an optical wave with the vibrational level of the material medium resulting in the rapid growth of the Stokes wave at the expense of the pump wave. Such an interaction has the reputation of generating stimulated emission at new wavelengths resulting in self frequency shifts giving rise to generation of short pulses at broader wave- lengths.

Much has been already dealt with forward stimulated Raman scattering which proves to possess the ability of turning optical fibers into broadband Raman amplifiers and tunable lasers (Agarwal 2001). We are interested in the rich dynamics of stimulated Raman back scattering (SRBS). SRBS observed in liquids (Maier et al. 1966) gives rise to the localized

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traveling backward scattered Stokes wave packet at the depletion of the forward propagating continuous wave pump. Also stimulated Raman back scattering in gases has proved itself to be a very significant route to compression and amplification of a Stokes seed-pulse by counter-propagating against a pump-pulse, as has been already established in various domains, mainly in free-space. The advent of hollow core photonic crystal fiber technology has made possible the practical realization of efficient Stokes and anti-Stokes emission, generation of Raman frequency combs (Abdolvand et al. 2009; Bauerschmidt et al. 2015) and infact the observation of efficient noise-seeded backward stimulated Raman scattering (Manoj et al. 2018).An appropriate model describing the dynamics of the SRBS process in the form of a resonant nonlinear three wave interaction between the pump wave, material wave and the back scattered Stokes wave has been constructed (Montes et al. 1997). The solitary solution of the three wave nonlinear partial differential equation governing the resonant SRBS has been derived and studied earlier (Picholle et al. 1991) and an extensive study of the process in the frame of a more general inertial model has also been carried out (Picozzi et al. 1998a). While the investigation of instability that arises on the top of Stokes pulse with modulus and phase of pump wave remaining constant is done with the damping of the pump neglected (Picozzi et al. 1998b), we present in this paper a general detailed investigation of nonlinear modulation instability (MI) of electromagnetic waves originating in the resonant three wave interaction with damping of the pump wave included. Though the resonant three wave interaction problem with bounded envelopes for the pump, Stokes and material wave has been analyzed quiet deeply, the MI analysis with the continuous pump wave poses a non-conservative complex problem and has not been much discussed. The notion behind the approach carried out here is to elucidate the impact of system parameters over the MI-induced dynamics in the semi-infinite media with continuous wave profiles for the three waves with due consideration of the pump, Stokes and material wave damping.

The universal MI was first theoretically analyzed by Hasegawa and Brinkman (1980) and first verified experimentally by Tai et al. (1986). This process provides a natural means of breakup of broad optical wave into train of ultra short pulses (Pelinovsky et al. 2000, 2002). Frequency generation, optical switching, laser techniques, such diverse applications of MI has attracted the attention of many researchers recently. Though this MI phenomenon which manifests itself as self-induced breakup of an initially homogeneous wave during its evolution in a nonlinear medium is found as a general feature in very many diverse fields, it is extensively studied in the field of optical fibers. Scalar MI was reported for the single pump wave propagating in a standard non birefringent fiber and in birefringent fiber the cross phase modulation (XPM) between two modes extends the instability do- main to the normal dispersion regime wherein the system involves more than one field component resulting in the vector MI. Various higher order linear and nonlinear effects such as higher order dispersions, self-steepening and time delayed Raman effects have also been found to strongly influence MI in fibers (Kivshar and Agrawal 2003; Ganapathy and Kuriakose 2002). The evolution of sidebands at a frequency separation from the carrier which is proportional to the square root of the optical pump power is the accompanying feature of MI (Seve et al. 1996). Long time evolution results in the growth of side bands near the fundamental wave leading to a mutual exchange of energy and hence the detailed study of growth of side bands proves beneficiary. We approach the MI phenomena that rules over the evolution of three wave resonant interaction of backward stimulated Raman process with the key perspective of studying the impact of detuning and nonlinear parameters over the side band evolution. We carry out the analysis exclusively for anomalous and normal regimes to enhance the practical realization of these phenomena in varied domains.

Our paper is structured as follows. In Sect. 2, the theoretical formulation of the problem is presented and the MI conditions both for the anomalous and nor- mal dispersion regimes are determined. In Sect. 3, the results of the numerical analysis are discussed and conclusions are made in the Sect. 4.

2 Theoretical formulation of the problem

The governing coupled mode equation for the resonant case of stimulated Raman scattering where a pump wave and its backscattered Stokes wave is coupled to a material response proportional to the molecular polarization is given by Picozzi et al. (1998b)

$$\begin{aligned}
&\left(\partial_t + \partial_z + i\rho\partial_{tt} + \mu_p\right)E_p = -E_sE_a + \kappa\left(\left|E_p\right|^2 + 2\left|E_s\right|^2\right)E_p,\\
&\left(\partial_t - \partial_z + i\rho\partial_{tt} + \mu_s\right)E_s = E_pE_a^* + \kappa\left(\left|E_s\right|^2 + 2\left|E_p\right|^2\right)E_s,\\
&\left(\left(1 + 2i\sigma\mu_a\right)\partial_t + \mu_a\right)E_a = E_pE_s^*,
\end{aligned} \tag{1}$$

where Ep, Es, Ea are the complex envelope amplitude of the pump wave, Stokes wave and the material response respectively and κ, σ, ρ are the Kerr, detuning and dispersion parameters respectively. For investigating the MI conditions, we assume the steady state continuous wave solutions of the form

$$\begin{aligned}
E_p &= \sqrt{P_p}\exp\left(i\phi_{pi}z\right),\\
E_s &= \sqrt{P_s}\exp\left(i\phi_{si}z\right),
\end{aligned} \tag{2}$$

Now Eq. (1) admits the steady state form Ea as $E_pE_s{}^*/\mu_a$, on substituting these steady state forms in Eq. (1) we obtain the phase factors as follows:

$$\phi_{pi} = \kappa\left(P_p + 2P_s\right), \phi_{si} = -\kappa\left(2P_p + P_s\right) \tag{3}$$

With $\mu_s = \frac{P_p}{\mu_a}$ and $\mu_p = -\frac{P_s}{\mu_a}$

The stability of the steady state solution is examined by looking into the system in the presence of small amplitude perturbations $a_p(z, t)$,$a_s(z, t)$,$a_a(z, t)$ given by

$$\begin{aligned}
E_p &= \left(\sqrt{P_p} + a_p[z,t]\right)\exp\left(i\phi_{pi}z\right),\\
E_s &= \left(\sqrt{P_s} + a_s[z,t]\right)\exp\left(i\phi_{si}z\right),\\
E_a &= \left(\sqrt{P_p}\sqrt{P_s}/\mu_a + a_a[z,t]\right)\exp\left(-i\phi_{si}z\right)\exp\left(i\phi_{pi}z\right)
\end{aligned} \tag{4}$$

Substituting Eq. (4) into the three coupled equations and on linearizing in the perturbation aj the following linearized equations are obtained

$$
\begin{aligned}
& i\frac{\partial a_p}{\partial t} + i\frac{\partial a_p}{\partial z} - \rho\frac{\partial^2 a_p}{\partial t^2} + \frac{\mathrm{i}\sqrt{P_p}P_s}{\mu_a} \\
& +\mathrm{i}\sqrt{P_p}\mu_p + \kappa P_p a_p + \mathrm{i}\mu_p a_p + 2\kappa\sqrt{P_p}\sqrt{P_s}a_s + \frac{\mathrm{i}\sqrt{P_p}\sqrt{P_s}a_s}{\mu_a} \\
& +\mathrm{i}\sqrt{P_s}a_a + \kappa P_p a_s + 2\kappa\sqrt{P_p}\sqrt{P_s}a_s^* = 0,
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
& i\frac{\partial a_s}{\partial t} - i\frac{\partial a_s}{\partial z} - \rho\frac{\partial^2 a_s}{\partial t^2} + -\frac{iP_p\sqrt{P_s}}{\mu_a} + i\sqrt{P_s}\mu_s + 2\kappa\sqrt{P_p}\sqrt{P_s}a_p \\
& - \frac{i\sqrt{P_p}\sqrt{P_s}a_p}{\mu_a} + \kappa P_s a_s + i\mu_s a_s + 2\kappa\sqrt{P_p}\sqrt{P_s}{a_p}^* + \kappa P_s {a_s}^* - i\sqrt{P_p}{a_a}^* = 0
\end{aligned}
\tag{6}
$$

$$
i\left(1 + 2i\sigma\mu_a\right)\frac{\partial a_a}{\partial t} + i\mu_a a_a - i\sqrt{P_s}a_p - i\sqrt{P_p}{a_s}^* = 0. \tag{7}
$$

In order to solve these set of linearized equations, we have assumed a plane wave ansatz constituting of both forward and backward propagation, having the form

$$
a_j = a_{jf}\exp[i(Kz - \Omega t)] + a_{jb}\exp[-i(Kz - \Omega t)] \tag{8}
$$

With f and b denoting forward and backward propagation. On substituting Eq. (8) into the linearized Eqs. (5)-(7) we get a set of 6 homogenous equations. These will have non-trivial solution only when the determinant of 6×6 stability matrix vanishes. ie.,

$$
MF = 0 \tag{9}
$$

The elements of non-zero matrix are

$$
M_{11} = -k + \Omega + \rho\Omega^2 + \kappa P_p + i\mu_p,
$$

$$
M_{12} = 2\kappa\sqrt{P_p}\sqrt{P_s} + \frac{i\sqrt{P_p}\sqrt{P_s}}{\mu_a},
$$

$$
M_{13} = i\sqrt{P_s},
$$

$$
M_{14} = \kappa P_p,
$$

$$
M_{15} = 2\kappa\sqrt{P_p}\sqrt{P_s},
$$

$$
M_{21} = \kappa P_p,
$$

$$
M_{22} = 2\kappa\sqrt{P_p}\sqrt{P_s}
$$

$$M_{23} = 0,$$

$$M_{24} = k - \Omega + \rho\Omega^2 + \kappa P_p + i\mu_p,$$

$$M_{25} = 2\kappa\sqrt{P_p}\sqrt{P_s} + \frac{i\sqrt{P_p}\sqrt{P_s}}{\mu_a},$$

$$M_{26} = i\sqrt{P_s},$$

$$M_{31} = 2\kappa\sqrt{P_p}\sqrt{P_s} - \frac{i\sqrt{P_p}\sqrt{P_s}}{\mu_a},$$

$$M_{32} = k + \Omega + \rho\Omega^2 + \kappa P_s + i\mu_s,$$

$$M_{33} = 0,$$

$$M_{34} = 2\kappa\sqrt{P_p}\sqrt{P_s},$$

$$M_{35} = \kappa P_s,$$

$$M_{36} = -i\sqrt{P_p},$$

$$M_{41} = 2\kappa\sqrt{P_p}\sqrt{P_s},$$

$$M_{42} = \kappa P_s,$$

$$M_{43} = -i\sqrt{P_p},$$

$$M_{44} = 2\kappa\sqrt{P_p}\sqrt{P_s} - \frac{i\sqrt{P_p}\sqrt{P_s}}{\mu_a},$$

$$M_{45} = -k - \Omega + \rho\Omega^2 + \kappa P_s + i\mu_s$$

$$M_{46} = 0,$$

$$M_{51} = -i\sqrt{P_s}$$

$$M_{52} = 0,$$

$$M_{53} = - = \Omega + i\mu_a + 2i\sigma\Omega\mu_a$$

$$M_{54} = 0,$$

$$M_{55} = -i\sqrt{P_p},$$

$$M_{56} = 0,$$

$$M_{61} = -i\sqrt{P_p},$$

$$M_{62} = 0,$$

$$M_{63} = -i\sqrt{P_s},$$

$$M_{64} = -i\sqrt{P_s},$$

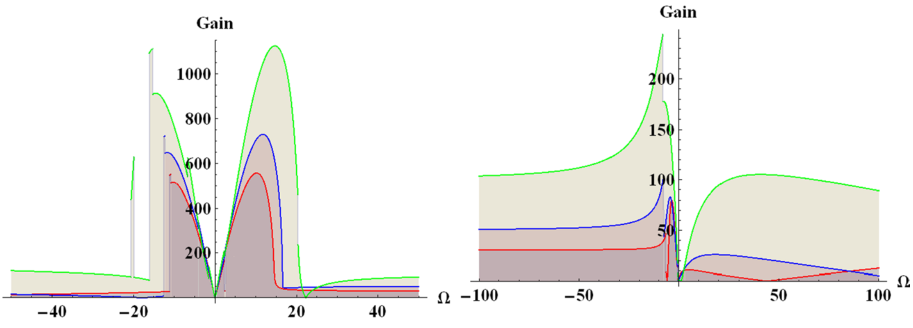
$$M_{65} = 0,$$

$$M_{66} = \Omega + i\mu_a - 2i\sigma\Omega\mu_a$$

To have the occurrence of MI there should be an exponential growth in the amplitude of the perturbed wave which implies the existence of a non vanishing imaginary part in the complex eigen value K. The measure of the gain is given by the parameter G = Im (K) where Im K means the imaginary part of K.

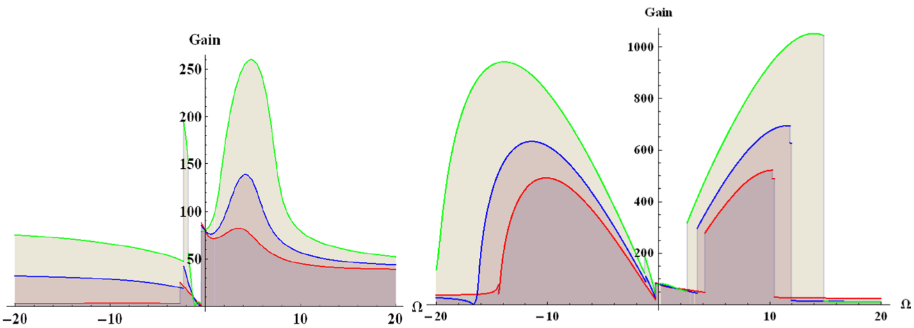
3 Results

The inertial model which we have considered here includes the nonlinear medium response along with the pump and Stokes wave. Apart from the conventional Benjamin-Feir instability, there is a possibility of inertial instability arising even in the absence of group velocity dispersion. At the outset we intend to obtain the surface plots of MI gain at possible combinations of Pump and Stokes power. Figure 1 Shows the MI gain plotted for different values of the pump power keeping the Stokes power constant. The obtained MI gain spectra in the anomalous regime is compared and contrasted with that of the normal regime as shown in Fig. 1. MI gain increases with pump power in both normal and anomalous regimes. But Instability gain is significantly larger in the normal regime than in the anomalous regime. Another predominant feature which is very much obvious in the first sight is the gain curves draining down to zero frequency in normal case and flattening edges sustaining at certain level of frequency in anomalous case.



(a) (b)

Fig. 1 Gain as a function of frequency Ω at fixed Stokes power Ps = 40 and increasing values of Pump power Pp = 30, 50 and 100 (red, blue and green) in the **a** normal dispersion region and **b** anomalous regime. The system parameters are $\sigma = 0.1$ and κ = 5



(a) (b)

Fig. 2 Gain as a function of frequency Ω at fixed pump power Pp = 40 and increasing values of Stokes power Ps = 30, 50 and 100 (red, blue and green) in the **a** normal dispersion region and **b** anomalous regime. The system parameters are $\sigma = 0.1$ and κ = 5

Further, as we tend to analyse the MI gain variation with that of Stokes power, the Pump power is chosen to have a constant value and the MI gain scenario is analysed at the increasing values of Stokes power. The obtained MI gain spectra in the anomalous regime is compared and contrasted with that of the normal regime as shown in Fig. 2. As the Stokes power is increased, the MI gain gets a hike too, as expected. But as we intrude it further, though the spectral window permitting MI is widened both in normal and anomalous regimes, it is seen that though the Stokes power is increased in the same manifold as that of the pump power in Fig. 1 case in both normal and anomalous regimes, there is a greater increase in the optimum gain value in the anomalous regime Fig. 2b than its counter part of Fig. 2a as in opposition to the Fig. 1 where normal regime permitted higher gain. One another significant difference seen in normal and anomalous case is that the rise and fall of the gain curves is distinct in the positive frequency domain of normal regime while in anomalous regime, the negative frequency domain shows typical MI gain side lobes.

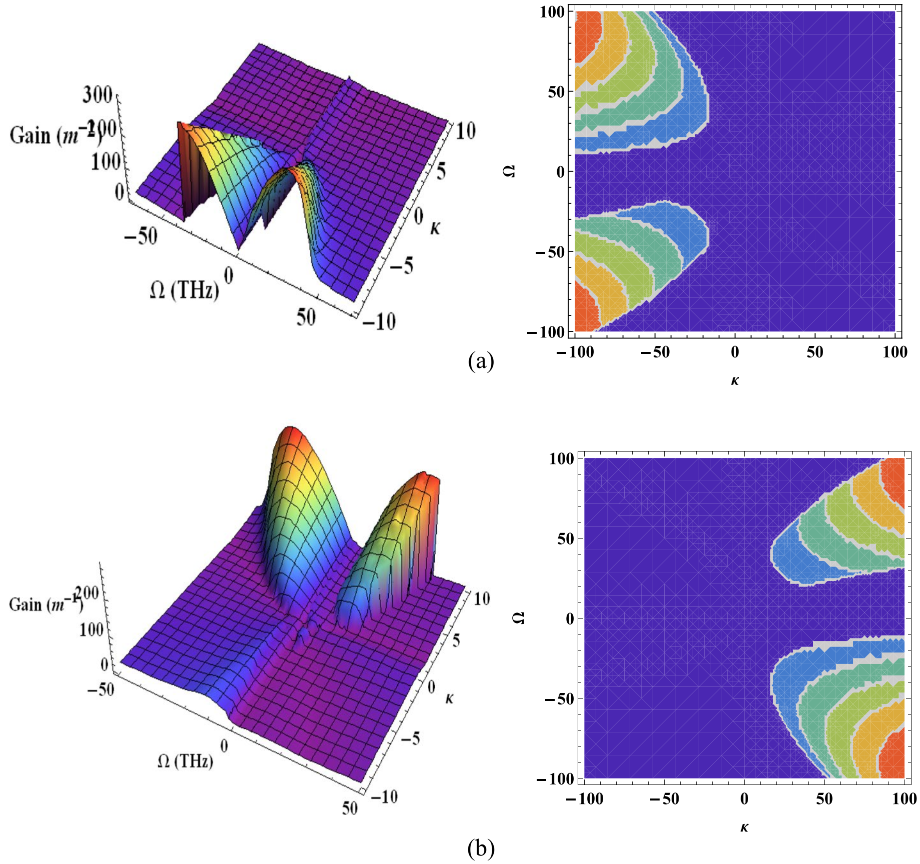


Fig. 3 Gain as a function of frequency Ω and Kerr parameter with fixed values of the Stokes power Ps and pump power Pp in the **a** normal dispersion regime and **b** anomalous dispersion regime with $\sigma = 0.2$. The corresponding contour plots shown alongside shows the increasing trend of MI in the defocusing case of nonlinearity in the normal regime and focusing case of nonlinearity in the anomalous regime

Both the Pump power variation and Stokes power variation is depicted for the focusing case of nonlinearity parameter ($\kappa > 0$). The behaviour seen in normal and anomalous regimes gets exactly reversed in the defocusing case of nonlinearity parameter ($\kappa < 0$).

The MI gain spectra is further analysed to throw light on the impact of the two crucial system parameters namely the Kerr parameter and the detuning parameter, the gain spectra plots compared in the normal and anomalous dispersion regimes explicitly. Figure 3a and b portray the gain plots as a function of frequency Ω and nonlinearity Kerr parameter κ. While normal regime shows MI lobes in the negative Kerr parameter regime i.e., the defocusing regime of nonlinearity $\kappa < 0$, anomalous plots depict the same scenario at the positive values (focusing case $\kappa > 0$) of Kerr parameter.

As we plot the gain as a function of detuning parameter α and frequency Ω in Fig. 4, an interesting scenario is observed. There are two sharp peaks of MI gain along the central frequency line near the zero detuning regimes. Infact there are four distinct cases underlying in the three dimensional portrayal of the gain spectra. Hence they are plotted explicitly in Fig. 5. We see different combinations of detuning parameter and frequency and we

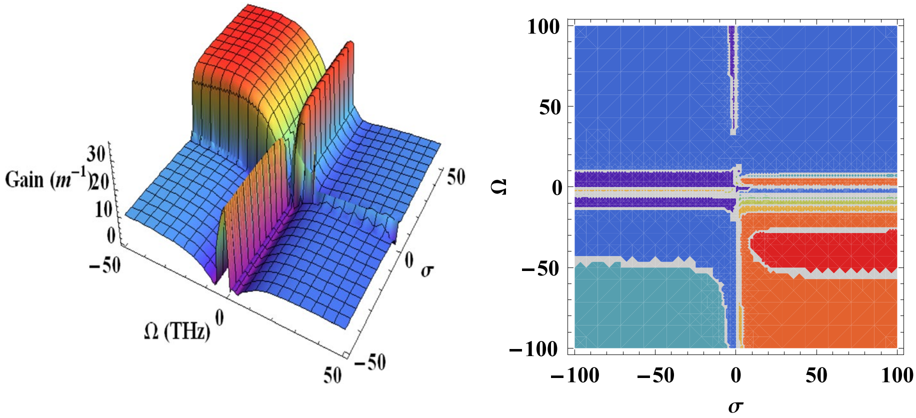


Fig. 4 3D Gain spectra as a function of frequency ω and detuning parameter σ with fixed values of the Stokes power Ps and pump power Pp along with the contour plot depiction

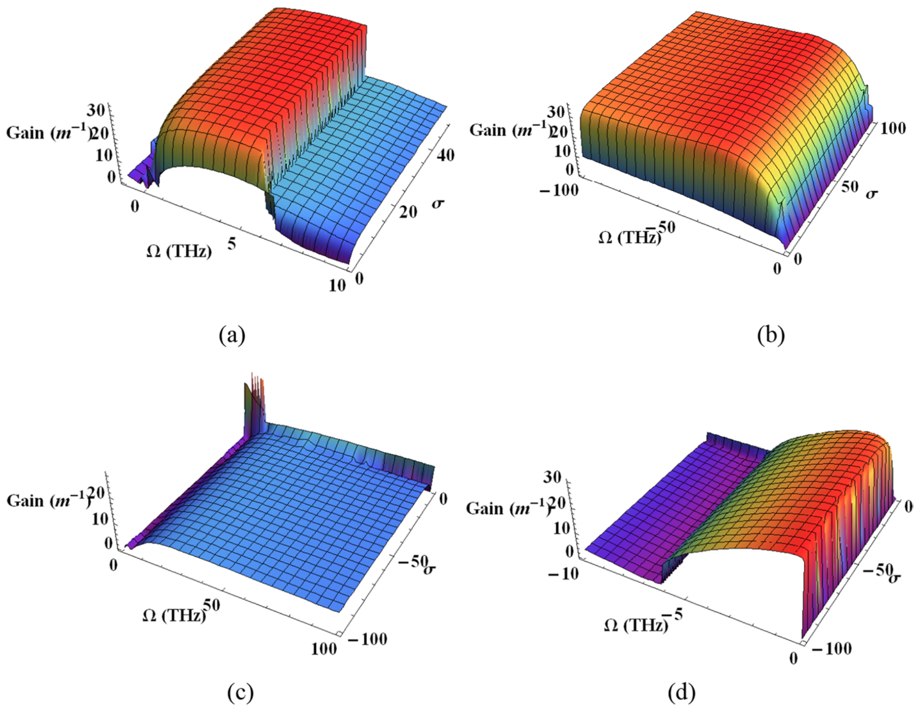


Fig. 5 3D gain shown explicitly in the four possible combinations of a) $\sigma > 0,\Omega > 0$, b) $\sigma > 0$,$\Omega < 0$, $c)\sigma < 0,\Omega > 0$, d) $\sigma < 0,\Omega < 0$. Of detuning parameter and frequency

immediately identify distinct MI gain. In the all positive regime, i.e., (a) $\sigma > 0,\Omega > 0$, there is a possible instability gain at low values of positive frequency regime between 0 and 5. In the (b) $\sigma > 0,\Omega < 0$, case, there is symmetrical flattened rise in the MI lobe throughout the regime. $(c)\sigma < 0,\Omega > 0$, case reveals a small peak along the central frequency line. And the (d) $\sigma < 0, \Omega < 0$. Case is just the counter part of case a where again distinct MI gain is observed at low values of negative frequency regime.

Note that we have arrived at MI gain analysis considering the medium to be very long, semi infinite, it could be said. However, one could consider a finite media, a possible amplifier configuration with proper boundary conditions as in Takushima and Kikuchi (1995). Considering the backscattering case, with pump field injected at z = 0, and Stokes field at z = L, L being the length of the fiber, the pump and Stokes field amplitude will no longer be constant, instead will be the functions of propagation distance and the stationary solutions will no longer be homogenous. In such a case, the system of linearized ordinary differential equations for the perturbations would involve spatially varying parameters and hence the stability analysis for propagation over a finite distance L should be performed by studying the modulus of the eigen values of the associated Jacobian matrix. Our future work would include this which would be of much practical relevance to amplifier applications.

4 Conclusion

In this paper, we have obtained the MI conditions for both anomalous and nor- mal dispersion regimes in a resonant case of stimulated Raman backscattering by plotting gain spectrum as function of Stokes power, Kerr parameter and detuning parameter. From these cases discussed above, where gain is plotted as a function of power, a distinct feature of side lobe formation critically depending upon the parameters is depicted. The characteristic feature of occurrence of inertial instability even in the no dispersion regime is portrayed and dependence of inertial gain over the system parameters is clearly depicted. Thus a detailed investigation on modulation instability has been done in the three wave resonant interaction of SRBS process.

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Author contributions AJ carried out the concept and design of the article. PM carried out data interpretation for the article. All authors read and approved the final manuscript.

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Data availability The data that has been used is confidential.

Declarations

Conflict of interest The authors declare no conflicts of interest.

Ethical approval Not applicable.

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