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Omnipresent instability criterion of birefringent Kundu-Eckhaus model with nonic nonlinearity

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ABSTRACT

This paper intends to present an investigation of the ultra shortpulse propagation under the nonlinear birefringent Kundu-Eckhaus (KE) model. The dynamics of modulational instability(MI) is revealed with the linear stability analysis of the KE equation with higher-order, the cubic, and nonic nonlinearity effects. The role of cubic-nonic nonlinearity on MI gain spectra in KE fiber system with self and cross phase modulation is studied for the first time. The impact of nonic nonlinearity is shown to be much more prominent over its cubic-quintic counterpart. The presence of cubic-nonic combo nonlinearity lets the MI gain profile to be adjusted with the rate of self-phase modulation (SPM). This new MI gain growth dynamics controlled by the SPM parameter will be useful on soliton generation in birefringent KE fiber media.

1. Introduction

A high-intensity optical beam propagating through a nonlinear dispersive medium leads to generation of various nonlinear effects. Among the different nonlinear effects, modulation instability (MI) is a process that stems from the interplay between nonlinear and dispersion effects and manifests itself as a natural means of breakup of an optical wave into a train of ultra-short pulses [1,2]. This process was first discovered in hydrodynamics [3] and further was shown to get exhibited in various fields such as fluid dynamics [4], plasma physics [5], dielectric media [6], electrodynamics [7], and atomic Bose-Einstein condensates [8,9], nonlinear optics. Theoretically, Hasegawa and Brinkman proposed an ultra-short pulse generation in glass optical fiber in an anomalous dispersion regime way back in 1984 [10]. Later, the generation of an ultrashort pulse is also shown to be possible in a normal dispersion regime with the inclusion of higher-order dispersion (HOD) coefficients [11,12]. The effect of fourth order dispersion (FOD) is shown to play a significant role in the generation of MI, leading to opening up new spectral windows [13]. *Tchofo* et al. studied the behavior of MI under the combined impacts of HOD and delayed Raman response analytically and quantitatively [14,15] and interestingly observed that the combination of HOD and delayed Raman response will suppress the MI in anomalous dispersion regime and opposite behavior is observed in normal dispersion regime. FOD is also proven to be capable of changing the MI peak gain to the higher frequency side and increase the instability region [16,17]. In standard silica-based optical fibers with Kerr nonlinearity, both self-phase modulation (SPM) and cross-phase modulation (XPM) induced MI process has been widely explored theoretically and experimentally [18–20]. Drummond et al. experimentally investigated the cross-phase MI with four-wave mixing in high birefringent fibers at the high-intensity laser pulses pumped under the Eigen states of polarization have a nonzero group velocity dispersion (GVD) mismatch [21]. Besides, the

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SPM-induced MI in semiconductor doped glass fiber with saturable nonlinearity was demonstrated in detail [22,23]. When a system is subjected to high pump power operation, often above the medium's saturation threshold, the saturation of nonlinearity becomes a strong case [24–27] and it showcases unique nonlinear features. The influence of the functional form of the SNL in supercontinuum generation is also studied in detail in Refs. [28–33], infact the different forms of SNL were analyzed for their impact in MI. In addition, *Inc* et al. have utilized a coupled nonlinear Schrödinger equation model to investigate the MI dynamics in monochromatic wave propagation through step-index optical fiber by standard linear stability analysis. To study the higher-order nonlinearity, the high intense sub-picosecond pulses are required for detailed studies. In 2012, MI dynamics were studied based on quintic nonlinearity, and the result shows that quintic nonlinearity plays a significant role in the stability of localized nonlinear structures. However, when the saturation is high, a self-focusing $\chi^{(7)}$ is also required [34]. With the help of spectrally resolved two beam coupling approach the fifth- and seventh-order nonlinearities of many glasses are studied experimentally and its shows that the important of seventh order nonlinearity will be used in multidimensional soliton generations [36]. The higher order nonlinear effects such as cubic, quintic, and septic nonlinearity drastically alter the properties of solition pulses and it mainly used in optical pulse compression or amplification, and solitary-wave-based communication networks, supercontinuum generation etc [35]. For the first time, *HouriaTriki* et al. studied the soliton generation with nonic nonlinearity in a unique background with various solitary waves like kink, dark, bright, and gray solitary are generated, which will not coexist the conventional nonlinear systems [36]. Furthermore, research into NLS models with higher-order nonlinearities is more essential than research into the more straightforward NLS equation. It's worth emphasizing that finding accurate solutions, particularly soliton solutions for higher-order NLS equations, is challenging and beneficial for optical communication systems.

The above-mentioned ultrashort pulse generations are entirely based on the nonlinear Schrödinger equation and modified nonlinear Schrödinger equation in optical fibers. Further versatile versions of extensive nonlinear equations like Lakshmanan–Porsezian–Daniel equation, the complex Ginzburg–Landau equation, the FokasLenells equation, and Kundu–Eckhaus (KE) equation [37–41] are also employed to study the nonlinear pulse propagation in nonlinear dispersive media. Among all, KE equations reveal a comprehensive range analytical solution similar to the nonlinear Schrödinger equation [42]. Using KE equations, the soliton dynamics in polarization preserving fibers, nano-fibers, and birefringence fibers are studied very well. *Vega-Guzman* et al. recently found the dark and solitary optical soliton solutions in birefringence media without the effect of four-wave mixing to the KE equation. *Yildirim* et al. modeled KE equation in coupled system using birefringent fiber without four wave mixing effect for the dark and singular soliton solutions with undetermined coefficients [43,44].

In this paper, as per our knowledge, for the first time, the KE equation with higher-order, the cubic, and nonic nonlinearity effects is considered and the MI dynamics have been studied under SPM and XPM parameters without four-wave mixing. In section-2, linear stability analysis is described. Section-3 explains the effect of nonic nonlinearity under SPM and XPM are explained in birefringent optical fiber. In section-5, the results of new findings have been concluded.

2. Theoretical model

To understand the cubic and nonic nonlinear effects, the dimensional model of the KE equation is modified by the following equation [33,34,45].

$$iZ_t + aZ_{xx} + b\left(|Z|^4 Z\right) + c\left(|Z|^2\right)_x Z = 0 \tag{1}$$

The first term depicts the evolution of the temporal pulse, the second term expands the group velocity dispersion coefficient, and parameters c and b represent nonlinear cubic and nonic, respectively. The KE Eq. (1) divides into the two components and equations listed below for studying the KE equation in birefringent fiber.

$$i\frac{\partial A}{\partial t} + u_1\frac{\partial^2 A}{\partial x^2} + \left(\alpha_1|A|^8 + \beta_1|A|^4|B|^4 + \gamma_1|B|^8\right)A + R_1\left(\frac{\partial\left(|A|^2\right)}{\partial x}\right)A + S_1\left(\frac{\partial\left(|B|^2\right)}{\partial x}\right)A = 0 \tag{2}$$

$$i\frac{\partial B}{\partial t} + u_2\frac{\partial^2 B}{\partial x^2} + \left(\alpha_2|B|^8 + \beta_2|B|^4|A|^4 + \gamma_2|A|^8\right)B + R_2\left(\frac{\partial\left(|B|^2\right)}{\partial x}\right)B + S_2\left(\frac{\partial\left(|A|^2\right)}{\partial x}\right)B = 0 \tag{3}$$

Where,A and B is slowly varying envelope, u_1 and u_2 is the group velocity dispersion parameters. While,SPM parameters are $\alpha_{1,2}$, $R_{1,2}$, and XPM terms are represented by$\beta_{1,2}$, $\gamma_{1,2}$ and $S_{1,2}$, respectively. The dynamics of optical soliton are described by Eq. (2) and (3) for birefringent optical fibers with SPM and XPM. We are considering all the critical points of SPM and XPM parameters under cubic and nonic nonlinearity. We study the mechanism of MI process under coupled KE equation in birefringent optical fibers, the stability analysis discussed in the following section.

2.1. Linear stability theory

The fundamental studies of MI process is dependent on linear stability analysis where the continues wave solution is perturbed by small quantum noise, the Eq. (2) and (3) have steady state solution in the manner of

$$A = a_0 \exp\left[i(k_1 x - \omega_1 t)\right] \tag{4}$$

$$B = b_0 \exp\left[i(k_2 x - \omega_2 t)\right] \tag{5}$$

While $k_1 = k_2 = k$ is the wavenumber, a_0 and b_0 are the flat wave's constant amplitude and angular frequency of wave is considered to be $\omega_1$andω_2. The frequency of values of ω_1 and ω_2 are given by the below equations and the values substituted in Eq. (4) and (5).

$$\omega_1 = u_1 k_1^2 - a_0^4 b_0^4 \eta_1 - a_0^8 \xi_1 - b_0^8 \zeta_1 \tag{6}$$

$$\omega_2 = u_2 k_2^2 - a_0^4 b_0^4 \eta_2 - b_0^8 \xi_2 - a_0^8 \zeta_2 \tag{7}$$

For verifying the stability of the continuous-wave steady state solution of Eq. (4) and (5) is investigated by analyzing the evolution of the system in the presence of a small complex perturbation,

$$A = (a_0 + \varepsilon_1[\mathrm{x},\mathrm{t}]) \exp\left[i(k_1 x - \omega_1 t)\right] \tag{8}$$

$$B = (b_0 + \varepsilon_2[\mathrm{x},\mathrm{t}]) \exp\left[i(k_2 x - \omega_2 t)\right] \tag{9}$$

Where, ε_1 and ε_2 are supposed to be modest perturbations in compared to the carrier wave amplitudes a_0 and b_0 of the following form

$$\varepsilon_1 = F_1 \exp[i(Qx - \Omega t)] + G_1 \exp[-i(Qx - \Omega t)] \tag{10}$$

$$\varepsilon_2 = F_2 \exp[i(Qx - \Omega t)] + G_2 \exp[-i(Qx - \Omega t)] \tag{11}$$

The complex conjugate is denoted by an asterisk. After linearizing with a perturbed equation ina unsettled ways, the following equations, be expressed in linear terms of ε_1, $\varepsilon_1{}^*$, ε_2, and $\varepsilon_2{}^*$,

$$\begin{aligned} & i\frac{\partial \varepsilon_1}{\partial t} + u_1 \frac{\partial^2 \varepsilon_1}{\partial \mathrm{x}^2} - k^2 u_1 \varepsilon_1 + 3a_0^4 b_0^4 \eta_1 \varepsilon_1 + 5a_0^8 \xi_1 \varepsilon_1 + \omega_1 \varepsilon_1 + b_0^8 \zeta_1 \varepsilon_1 + 2a_0^4 b_0^4 \eta_1 \varepsilon_1{}^* + 4a_0^8 \xi_1 \varepsilon_1{}^* + 2a_0^5 b_0^3 \eta_1 \varepsilon_2 + 4a_0 b_0^7 \zeta_1 \varepsilon_2 + 2a_0^5 b_0^3 \eta_1 \varepsilon_2{}^* \\ & \quad + 4a_0 b_0^7 \zeta_1 \varepsilon_2{}^* + 2iku_1 \frac{\partial \varepsilon_1}{\partial t} + R_1 a_0^2 \frac{\partial \varepsilon_1}{\partial t} + R_1 u_0^2 \frac{\partial \varepsilon_1{}^*}{\partial t} + S_1 a_0 b_0 \frac{\partial \varepsilon_2}{\partial t} + S_1 a_0 b_0 \frac{\partial \varepsilon_2{}^*}{\partial t} = 0 \end{aligned} \tag{12}$$

$$\begin{aligned} & i\frac{\partial \varepsilon_2}{\partial t} + u_2 \frac{\partial^2 \varepsilon_2}{\partial \mathrm{x}^2} + 2a_0^3 b_0^5 \eta_2 \varepsilon_1 + 4a_0^7 b_0 \zeta_2 \varepsilon_1 + 2a_0^3 b_0^5 \eta_2 \varepsilon_1{}^* + 4a_0^7 b_0 \zeta_2 \varepsilon_1{}^* - k^2 u_2 \varepsilon_2 + 3a_0^4 b_0^4 \eta_2 \varepsilon_2 + 5b_0^8 \xi_2 \varepsilon_2 + \omega_2 \varepsilon_2 + a_0^8 \zeta_2 \varepsilon_2 + 2a_0^4 b_0^4 \eta_2 \varepsilon_2{}^* \\ & \quad + 4b_0^8 \xi_2 \varepsilon_2{}^* + S_2 a_0 b_0 \frac{\partial \varepsilon_1}{\partial t} + S_2 a_0 b_0 \frac{\partial \varepsilon_1{}^*}{\partial t} + 2iku_2 \frac{\partial \varepsilon_2}{\partial t} + R_2 b_0^2 \frac{\partial \varepsilon_2}{\partial t} + R_2 b_0^2 \frac{\partial \varepsilon_2{}^*}{\partial t} = 0 \end{aligned} \tag{13}$$

We get a set of four linearly connected equations if we replace Eqs. (10) and (11) with Eqs. (6) and (7). Only if the 4 × 4 determinant created by the matrix of coefficients vanishes, as shown below, a non-trivial solution is possible:

$$\begin{pmatrix} \Omega + A_1 & C_1 & B_1 & D_1 \\ B_2 & -\Omega + A_2 & C_2 & D_2 \\ C_3 & D_3 & \Omega + A_3 & B_3 \\ C_4 & D_4 & B_4 & -\Omega + A_4 \end{pmatrix} \begin{pmatrix} F_1 \\ G_1 \\ F_2 \\ G_2 \end{pmatrix} = 0 \tag{14}$$

In the appendix, the coefficients of matrix elements are appended in detail.The vanishing determinant of the matrix results in the following condition

$$\Omega^4 + \Omega^3 \mathrm{L} + \Omega^2 \mathrm{M} + \Omega \mathrm{N} + \mathrm{O} = 0 \tag{15}$$

Where,

$$L = (A_1 - A_2 + A_3 - A_4);$$

$$M = (-A_1A_2 + A_1A_3 - A_2A_3 - A_1A_4 + A_2A_4 - A_3A_4 + B_2B_3 + B_1C_2 - C_1C_3 + B_4D_1 + C_4D_3 - D_2D_4);$$

$$\begin{aligned} N = & (-A_1A_2A_3 + A_1A_2A_4 - A_1A_3A_4 + A_2A_3A_4 + A_1B_2B_3 - A_4B_2B_3 + A_3B_1C_2 - A_4B_1C_2 - B_3C_1C_2 - B_1B_2C_3 + A_2C_1C_3 + A_4C_1C_3 \\ & - A_2B_4D_1 + A_3B_4D_1 - C_3C_4D_1 + D_2B_1B_4 + D_2B_3C_4 - D_3C_1B_4 + D_3A_1C_4 - D_3A_2C_4 + D_3D_1C_2 - D_2A_1D_4 - D_2A_3D_4 + D_3B_3D_4); \end{aligned}$$

$$\begin{aligned} O = & (A_1A_2A_3A_4 - A_1A_4B_3B_2 - A_3A_4B_3C_2 + A_4B_3C_1C_2 + A_4B_1C_3B_2 - A_2A_4C_3C_1 - A_2A_3B_4D_1 + B_2B_3B_4D_1 - C_2B_3C_4D_1 + C_2A_2C_4D_1 \\ & + B_2A_3B_4D_2 - B_2C_1B_4D_2 + B_2A_1C_4D_2 - B_1C_3B_4D_2 - B_2B_1B_4D_2 + C_2A_2B_4D_3 - A_2A_1B_4D_3 + B_2C_3C_4D_3 + C_2A_3D_4D_2 - B_2C_3D_4D_2 \\ & - A_2A_3D_4D_2 + C_2C_3D_4D_2 + B_2A_3D_4D_2 - C_2C_3D_4D_2); \end{aligned}$$

We solve Eq. (15), we obtain the dispersion relationship

$$\begin{aligned}\Omega = &\pm\frac{1}{4}\Big(-A_1+A_2-A_3+A_4\Big)-\frac{1}{2}\sqrt{\Big(A_1A_2-A_1A_3+A_3A_2+\frac{1}{4}(-A_1+A_2-A_3+A_4)^2+A_1A_4-A_4A_2+A_4A_3-B_3B_2-B_1C_2}\\
&+C_3C_1-B_4D_1-C_4D_3+D_4D_2+\frac{1}{3}(-A_1A_2-A_1A_3+A_3A_2-A_1A_4-A_2A_4+A_3A_4+B_3B_2+B_1C_2+C_3C_1-B_4D_1-C_4D_3+D_4D_2)\\
&+\Big(2^{\frac{1}{3}}((-A_1A_2-A_1A_3-A_3A_2-A_1A_4-A_2A_4+A_3A_4+B_3B_2+B_1C_2+C_3C_1-B_4D_1-C_4D_3+D_4D_2)^2\\
&-3(A_1+A_2-A_3+A_4)(-A_1A_2A_3+A_1A_2A_4-A_1A_4A_3+A_1A_2A_3+A_1B_2B_3-A_4B_2B_3+A_3B_1C_3-A_4B_1C_3-B_3C_1C_3-B_2B_1C_3\\
&+A_2C_1C_3+A_4C_1C_3-A_2B_4D_1+A_3B_4D_1-C_3B_4D_1+B_1B_4D_2+B_3C_1C_3-B_2B_1C_3+A_2C_1C_3+A_4C_1C_3-A_2B_4D_1+A_3B_4D_1\\
&-C_3B_4D_1+B_1B_4D_2)+12(A_1A_2A_3A_4-A_1B_2B_3A_4-A_3B_4C_3D_4+A_4B_2B_3C_4+A_3B_4C_3D_4+A_4D_2C_3C_4-A_3B_4C_3D_4+C_4B_2B_3D_4\\
&+A_1C_4C_3D_1+A_2D_2B_3C_1+A_3B_4C_3D_4+D_4A_2A_3C_2-B_3B_4D_3D_4+A_4D_2C_3C_4-A_3B_4C_3D_4+C_4B_2B_3D_4-A_1C_4C_3D_1+A_2D_2B_3C_1\\
&-C_1B_4C_3D_4+C_4B_2B_3D_4+A_1C_4C_3D_1+A_2D_2B_3C_1+A_3B_4C_3D_4+A_4B_2B_3C_4))\Big);\end{aligned} \tag{16}$$

There is a real and imaginary values of Ω are obtained in Eq. (16). The imaginary component of a disturbing frequency defines the modulation rate of wave number Q. In contrast, the real part of the Ω is represented as the frequency shift compared to the uniform model. The imaginary portion thus determines the stability of the birefringent KE equation of nonlinear optical wave propagation. The KE equation birefringent fiber MI criteria can be defined as

$$G(\Omega) = |Im(\Omega_{\max})| \tag{17}$$

3. Modulational instability characteristics

To provide an overview of the picture, a detailed mathematical treatment for the process of MI dispersion relation (see Eq.17) corresponding to the coupled birefringent optical fiber media with the effect of cubic and nonic nonlinearity are solved in the above section detail. Followed by the dispersion relation, MI gain spectrum obtained with the inclusion of SPM, XPM, cubic and nonic nonlinearity terms, and the generated MI gain spectrum plotted in Figs. 1 and 2. To study the MI growth rate and stability/instability nature of the MI process, the nonlinear XPM($\beta_{1,2}$,$\gamma_{1,2}$,$S_{1,2}$) and SPM ($\alpha_{1,2}$, $R_{1,2}$)values are varied in a birefringent KE fiber system. As a result, the gain spectrum profile as a function of Q and k is shown in Figs. 1 and 2, where the orange color region indicates that the generated MI waves are having more stability with regard to modulation of any wave number Q and the bright red region indicates that the wave is unstable in nature.In order to analyze the influence of XPM and SPM on optical wave propagation in coupled birefringent fiber: In the first case role of SPM in coupled birefringent fiber with the fixed XPM parameter is analyzed and the second scenario involves adjusting the XPM while keeping the SPM parameter constant.

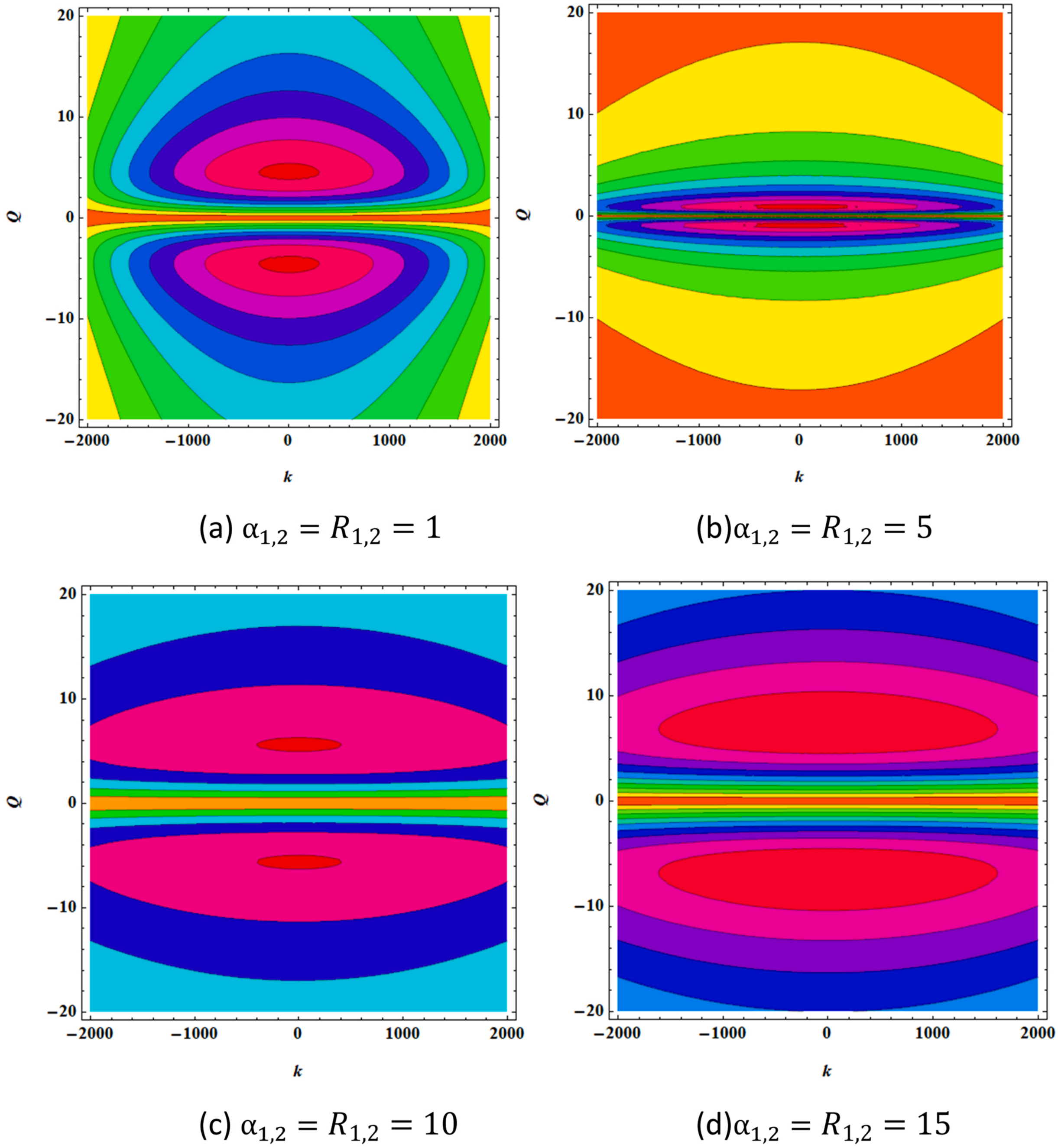
3.1. The impact of SPM at constant XPM

In this section, the influence of SPM parameters at constant XPM under the higher-order cubic and nonic, nonlinearities is discussed in detail. Initially, the value of SPM ($\alpha_{1,2}$, $R_{1,2}$) is fixed at 1. Fig. 1(a) shows that the generated MI process as a function of Q and k, the stable zone is most significant over the frequency range and the unstable region is lowest as the SPM parameter value is increased from 5 to 15 with an increment of 5. Interestingly, the domain size of the stability region starts reducing when the SPM parameter was increasing. Subsequently, the unstable region started growing with frequency, as shown in Fig. 1(b)– (d). This means, the SPM parameter plays significant role on MI dynamics for soliton generation.

3.2. The impact of XPM at constant SPM

On the other hand, for understanding the impact of XPM parameters, the SPM is set to be constant, and XPM parameters is varied from the values of 1.2–5.2 with an increment of 1. Here we observed that the growth of the MI process is entirely different behavior in nature as compared to SPM dynamics. Here the XPM parameters dominate in the development of the stability region. Moreover, increasing the XPM parameters above 3.2, the instability regime shrunk over the frequency range as well the instability regime appeared very far from the zero-dispersion wavelength. Tuning the XPM parameter causes the generation of solitons. The instability to stability decrement is portrayed clearly in Fig. 2(a)– (d). In this study, we identified that SPM parameters cause more instability in the birefringent fiber of the KE equation than XPM parameter values. The instability to stability tuning by SPM and XPM under cubic and nonic nonlinearities might prove very helpful in soliton switching applications in the future.

We now switch to investigate the amplitude growth profile of MI process in coupled birefringent KE equation by varying SPM and XPM parameters under cubic and nonic nonlinearity. The maximum amplitude growth is indicated in red, and the minimum amplitude growth is marked in orange in the Figs. 3 and 4. As per the two conditions, we observed the amplitude change is precisely the same in



(a) $\alpha_{1,2} = R_{1,2} = 1$ (b) $\alpha_{1,2} = R_{1,2} = 5$

(c) $\alpha_{1,2} = R_{1,2} = 10$ (d) $\alpha_{1,2} = R_{1,2} = 15$

Fig.:1. (Contour plot) Modulational instability gain spectrum of coupled birefringent KE model in the presence of various strength of cubic-nonic nonlinearity. Remaining nonlinear parameters are $a_0 = 0.1$, $b_0 = 5.8$, $u_{1,2} = 0.2$, $\beta_{1,2} = \gamma_{1,2} = 1.2$ and $S_{1,2} = 1.2$.

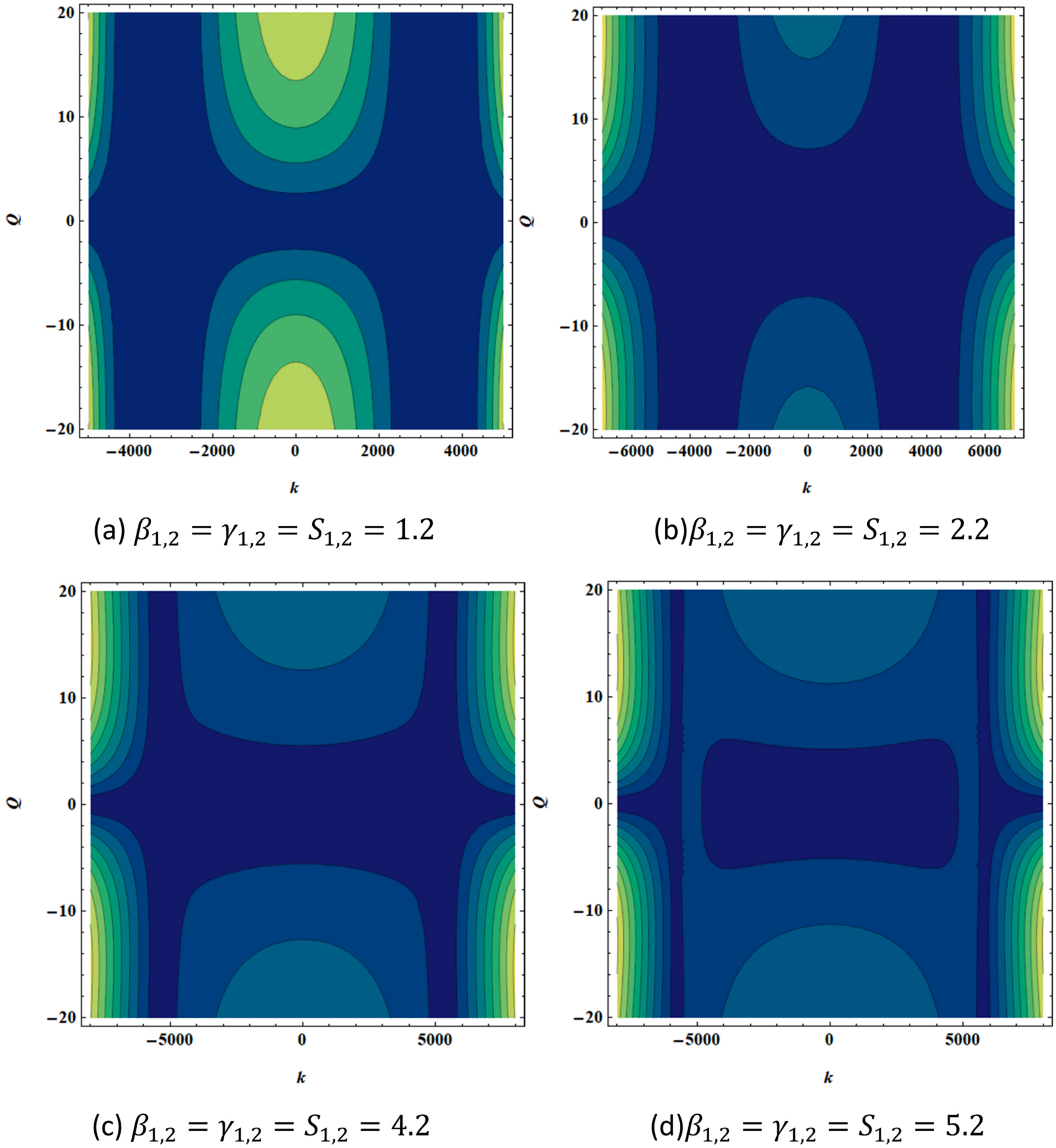


Fig.:2. (Contour plot) Modulational instability gain spectrum of coupled birefringent KE model with help of different cubic-nonic nonlinear strength. Other nonlinear parameters are $a_0 = 0.1$, $b_0 = 5.8$, $u_{1,2} = 0.2$, $\alpha_{1,2} = 1$ and $R_{1,2} = 1$.

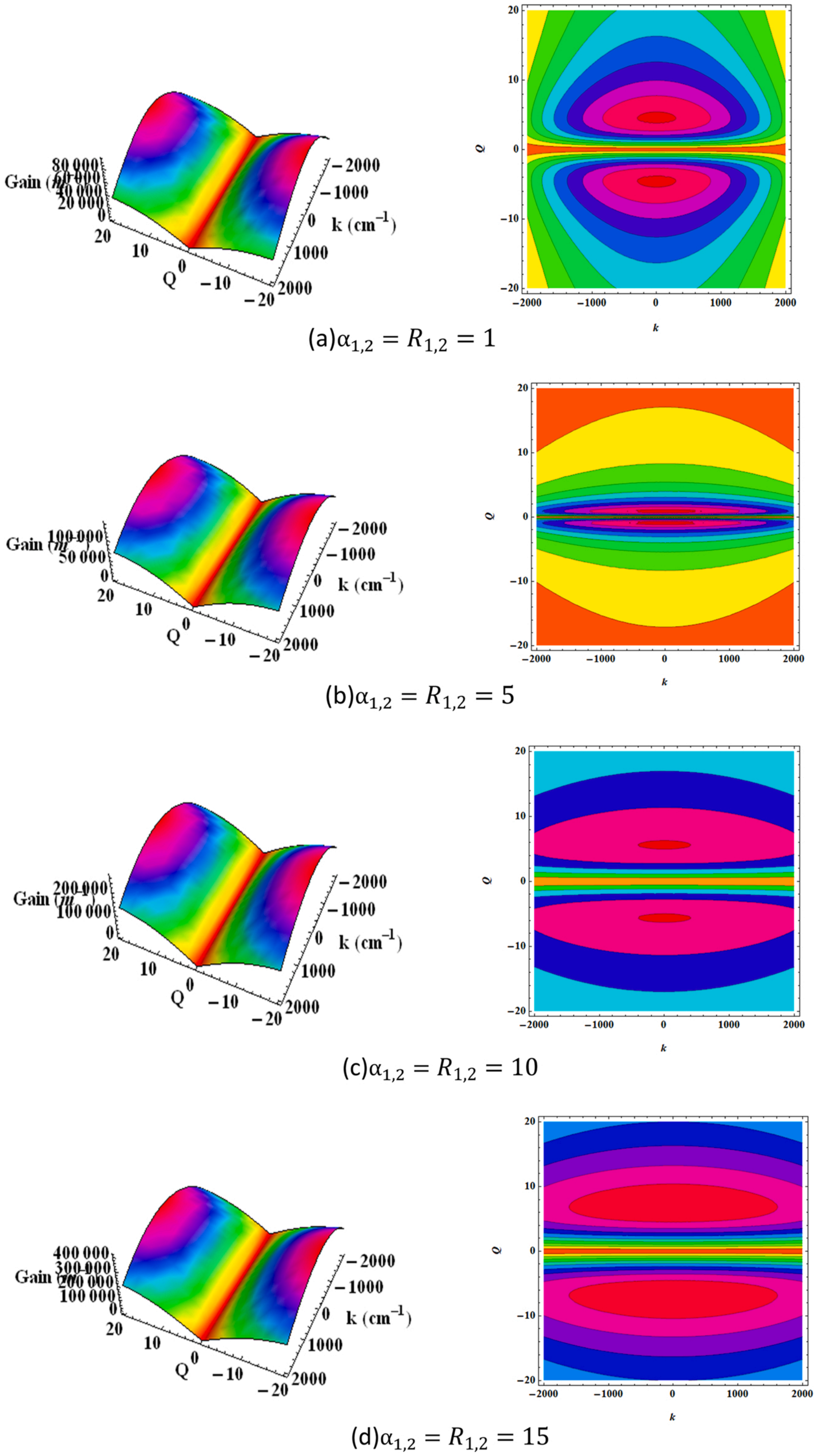


Fig.: 3. (Contour plot with 3D figure) Depicts the variation of instability gain profile of coupled birefringent KE model in the presence of cubic-nonic nonlinearity. Remaining nonlinear parameters are $a_0 = 0.1$, $b_0 = 5.8$, $u_{1,2} = 0.2$, $\beta_{1,2} = \gamma_{1,2} = 1.2$ and $S_{1,2} = 1.2$.

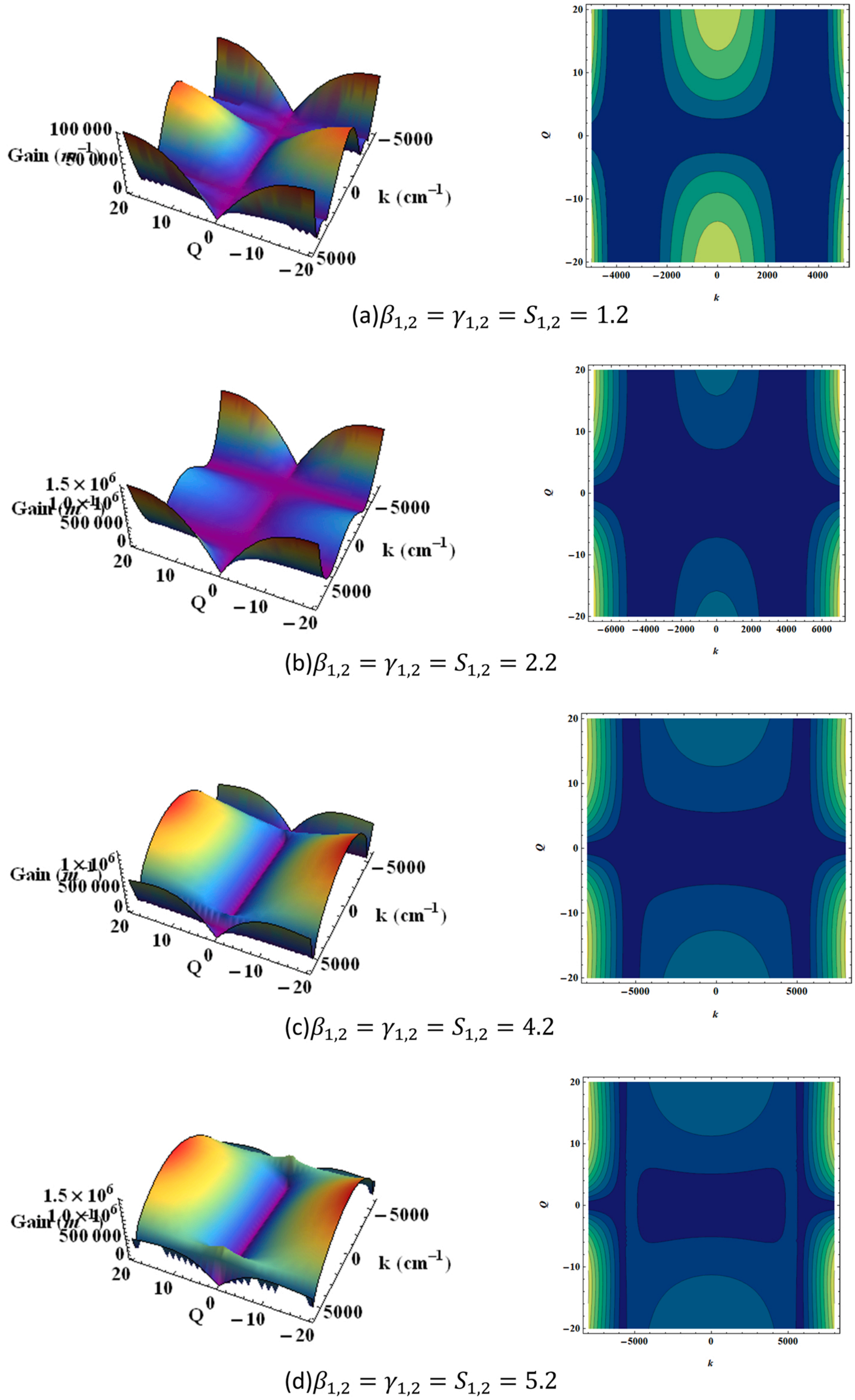


Fig.: 4. (Contour plot with 3D figure) Shows the increasing trend of Modulational instability gain spectrum of coupled birefringent KE model with adjustable strength of cubic-nonic nonlinearity. Remaining nonlinear parameters are $a_0 = 0.1$, $b_0 = 5.8$, $u_{1,2} = 0.2$, $\beta_{1,2} = \gamma_{1,2} = 1.2$ and $S_{1,2} = 1.2$.

the previous section. In condition1, the XPM is constant, and SPM is varied from 5 to 15 with an increment of 5, the size of the mode at the corner mode decreases, and the expansion of the mode at the center appears to expand near the highest value of SPM. The illustration of the MI amplitude profile is shown in Fig. 3(a)-(d). In condition2, by tuning XPM, the amplitude growth increases even in the instability region (see Fig. 4(a)-(d)). This amplitude growth mainly occurs even in the MI process side lobes generated at very far from the zero-dispersion regime.

4. Conclusions

In this paper, the MI criterion of coupled birefringent fiber of the KE model with SPM and XPM was investigated under cubic and nonic nonlinearity profile without a four-wave mixing term. Using linear stability analysis, the KE equation model dispersion relation is obtained for the generation of MI. When the tuning of SPM and XPM parameters under cubic and nonic nonlinearity is carried out, as SPM values at constant XPM is increased, the number of MI side lobes are constant with increasing trend of gain profile. But, in the case of XPM variation keeping SPM value constant, interestingly the generation of side lobes also increases with the increased XPM values as clearly shown in 3D graph. On the other side, adjusting the XPM parameter at constant SPM, the stability region gets increased with increasing XPM strength and with constant XPM, tuning SPM to higher values decreases the stability region as depicted in the Contour plots. This dynamical behavior of the MI process with the growth amplitude and stability to instability changes under SPM and XPM effects could be adopted for soliton switching applications. In the future, the impact of four-wave mixing along with nonic nonlinearity will be studied.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

$$A_1 = k^2a_1 - 2kQa_1 - Q^2a_1 + iQr_1u_0^2 + 3u_0^4v_0^4\eta_1 + 5u_0^8\xi_1 + \omega_1 + v_0^8\zeta_1;$$

$$A_2 = k^2a_2 - 2kQa_2 - Q^2a_2 + iQr_2v_0^2 + 3u_0^4v_0^4\eta_2 + 5v_0^8\xi_2 + \omega_2 + u_0^8\zeta_2;$$

$$A_3 = k^2a_1 - 2kQa_1 - Q^2a_1 + iQr_1u_0^2 + 3u_0^4v_0^4\eta_1 + 5u_0^8\xi_1 + \omega_1 + v_0^8\zeta_1;$$

$$A_4 = k^2a_2 - 2kQa_2 - Q^2a_2 + iQr_2v_0^2 + 3u_0^4v_0^4\eta_2 + 5v_0^8\xi_2 + \omega_2 + u_0^8\zeta_2;$$

$$B_1 = iQs_1u_0v_0 + 2u_0^5v_0^3\eta_1 + 4u_0v_0^7\zeta_1;$$

$$B_2 = -iQs_1u_0v_0 + 2u_0^5v_0^3\eta_1 + 4u_0v_0^7\zeta_1;$$

$$B_3 = iQs_1u_0v_0 + 2u_0^5v_0^3\eta_1 + 4u_0v_0^7\zeta_1;$$

$$B_4 = -iQs_1u_0v_0 + 2u_0^5v_0^3\eta_1 + 4u_0v_0^7\zeta_1;$$

$$C_1 = -iQr_1u_0^2 + 2u_0^4v_0^4\eta_1 + 4u_0^8\xi_1;$$

$$C_2 = -iQr_2v_0^2 + 2u_0^4v_0^4\eta_2 + 4v_0^8\xi_2;$$

$$C_3 = iQr_1u_0^2 + 2u_0^4v_0^4\eta_1 + 4u_0^8\xi_1;$$

$$C_4 = iQr_2v_0^2 + 2u_0^4v_0^4\eta_2 + 4v_0^8\xi_2;$$

$$D_1 = \mathbb{i}Qs_2u_0v_0 + 2u_0^3v_0^5\eta_2 + 4u_0^7v_0\zeta_2;$$

$$D_2 = -\mathbb{i}Qs_2u_0v_0 + 2u_0^3v_0^5\eta_2 + 4u_0^7v_0\zeta_2;$$

$$D_3 = \mathbb{i}Qs_2u_0v_0 + 2u_0^3v_0^5\eta_2 + 4u_0^7v_0\zeta_2;$$

$$D_4 = -\mathbb{i}Qs_2u_0v_0 + 2u_0^3v_0^5\eta_2 + 4u_0^7v_0\zeta_2;$$

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